Analytical approach for Francis turbine part load resonance risk assessment

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Abstract. Francis turbines operating at part load conditions experience cavitating vortex rope in the draft tube resulting from the swirling flow at the runner outlet. This cavitating vortex rope induces convective and synchronous pressure fluctuations at the rope precession frequency. The pressure fluctuating synchronous component, comprised between 0.2 and 0.4 times the turbine rotational speed, can be addressed as a draft tube pressure source forced excitation of the entire hydraulic system including the turbine itself. The synchronous component propagates through the entire hydraulic circuit and may lead to hydroacoustic resonance if the part load excitation frequency matches with one of the hydraulic system natural frequencies or even the natural frequencies of the synchronous generator. The paper presents a simplified analytical method to assess the resonance risk of the hydraulic system at early stage of a project. This method is based on a first estimation of the hydraulic system natural frequencies which is achieved from hydroacoustic properties of the hydraulic system such as pipe length, cross section area and wave speed. The cavitating draft tube is modelled with equivalent wave speed representative of the cavitation compliance and with a hydraulic inductance representative to the draft tube water inertia. The accuracy of this method is evaluated by comparison with a detailed 1D SIMSEN software frequency analysis, enabling to determine the eigen frequencies and eigen mode shapes of the hydraulic system, considering 3 different Francis turbine hydraulic layouts in terms of tailrace tunnel's length and diameter. The simplified methodology provides reasonably good results to identify potential risk of resonance at early stage of the project. The proposed analytical method for the assessment of the Francis turbine part load resonance risk is nowadays included as ANNEXE E.3 of the technical specification of the new IEC Technical Specification 62882 ED1 entitled "Hydraulic machines – Francis turbine pressure fluctuation transposition" which was issued in 2020-09.

1. Introduction

Unless extreme submergence applies, Francis turbines operating at part load conditions experiences cavitating vortex rope in the draft tube resulting from the swirling flow at the runner outlet, [4], [5], [7]. This cavitating vortex rope induces convective and synchronous pressure fluctuations at the rope precession frequency, [3]. The pressure fluctuating synchronous component, comprised between 0.2 and 0.4 times the turbine rotational speed, can be addressed as a draft tube pressure source forced excitation of the entire hydraulic system including the turbine itself, [4], [18]. The synchronous component propagates through the entire hydraulic circuit and may lead to hydroacoustic resonance if the part load excitation frequency matches with one of the hydraulic system natural frequencies or even the natural frequency of the synchronous generator, [14]. At early stage of a project, it is useful to evaluate if such vortex rope resonance may occur on the prototype, in order to anticipate the installation of possible mitigate measures such as air injection (central or peripheral), fins in the draft tube, draft tube with central column, runner cone extension, PSS parameter special tuning, see [18]. To this end, one may first use very basic analytical approach to assess the risk of hydroacoustic part load resonance by estimating the main natural frequencies of the penstock. Figure 1 depicts schematic representation of the elastic pressure mode shapes of the 3 first natural frequencies of the penstock, which can be evaluated for any order *k* as follows:

$$f_k = \frac{a_1}{\lambda_k} = \frac{a_1}{4 \cdot l_1}(2 \cdot k - 1) \quad ; \quad k = 1, 2, 3... \qquad (1)$$

Where:

a_1: mean wave speed of the penstock (m/s)

l_1: length of the penstock (m)

f_k: penstock natural frequency of order *k* (Hz)

λ_k: wave length of mode shape of order *k* (m)

As mentioned, the draft tube vortex rope dominant excitation frequency can be estimated as follows:

$$f_{exc\_rope} = 0.2 - 0.4 \cdot f_n \qquad (2)$$

Therefore, hydroacoustic resonance between the cavitating vortex rope and the hydraulic system may occur if the vortex rope excitation frequency matches with one the of the first 6 penstock natural frequencies, which may be expressed with the following condition:

$$f_{exc\_rope} = f_k \quad ; \quad k \in [1, 2, .., \sim 6] \qquad (3)$$

It is expected that resonance with penstock natural frequencies higher that the 6th order, are subject to sufficient damping to prevent excessive pressure fluctuation amplitudes.

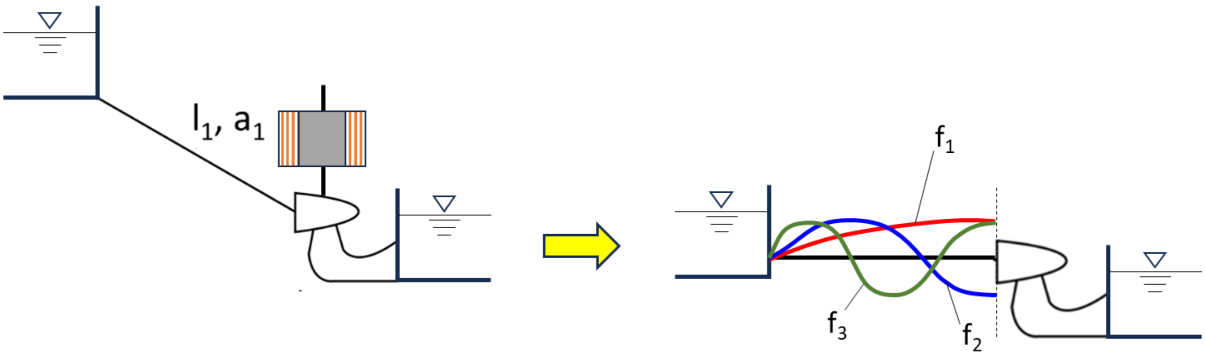


Figure 1. Penstock elastic pressure mode shapes for its 3 first natural frequencies.

In addition, considering the vortex rope excitation frequency of equation (2) and knowing that synchronous generators usually feature the so-called electromechanical mode of oscillation, also known as the “generator natural frequency” in the range between 0.7 Hz and 2 Hz, see [16], possible resonance between the cavitating vortex rope and the synchronous generator can be assessed with the Figure 2 based on the knowledge of the unit nominal rotational speed.

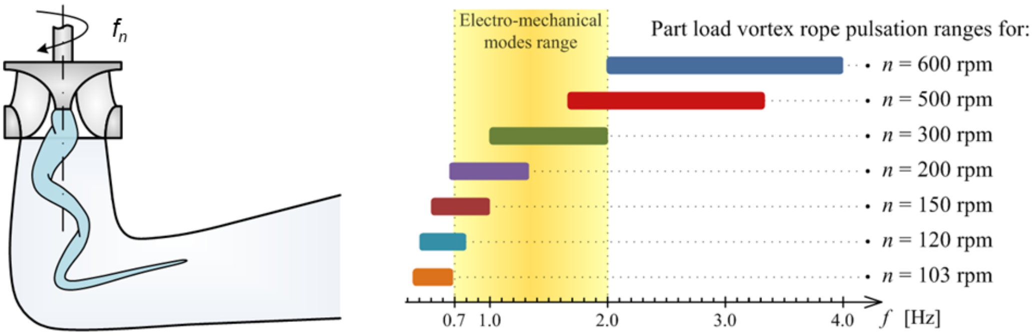


Figure 2. Synchronous generators nominal rotational speed n which may present a risk of resonance with the cavitating draft tube vortex rope, adapted from [16].

The above simplified resonance risk assessment approach has the great advantage that it requires only few data, which can be fairly estimated at the early stage of a project. However, this approach does not take into account the influence of the hydroacoustic parameters of the cavitating draft tube itself, nor of the possible tailrace tunnel. Therefore, additional simplified analytical models taking into account the possible influence of the cavitating draft tube itself and of the tailrace tunnel available in the literature are summarized in this paper. The accuracy of these models is then evaluated by comparison with the results of eigenvalue and eigen mode detailed calculation performed with the SIMSEN software for 3 different typical Francis turbine hydraulic layouts.

2. 1D detailed hydroacoustic modelling

By assuming uniform pressure and velocity distributions in the cross section and neglecting the convective terms, the one-dimensional momentum and continuity balances for an elementary pipe filled with water of length *dx*, cross section *A* and wave speed *a*, see Figure 3, yields to the following set of hyperbolic partial differential equations, see Wylie and Streeter, [17]:

$$\begin{cases} \dfrac{\partial h}{\partial t} + \dfrac{a^2}{gA} \cdot \dfrac{\partial Q}{\partial x} = 0 \\ \dfrac{\partial h}{\partial x} + \dfrac{1}{gA} \cdot \dfrac{\partial Q}{\partial t} + \dfrac{\lambda |Q|}{2gDA^2} \cdot Q = 0 \end{cases} \tag{4}$$

This system is solved using the Finite Difference Method with a 1st order centred scheme discretization in space and a scheme of Lax for the discharge variable. This approach leads to a system of ordinary differential equations that can be represented as a T-shaped equivalent scheme [8], [13], [11]. The RLC parameters of this equivalent scheme are given by:

$$R = \frac{\lambda \cdot |\overline{Q}| \cdot dx}{2 \cdot g \cdot D \cdot A^2} \qquad L = \frac{dx}{g \cdot A} \qquad C = \frac{g \cdot A \cdot dx}{a^2} \tag{5}$$

Where λ is the local loss coefficient and D is the diameter of the elementary pipe. The hydraulic resistance R, the hydraulic inductance L and the hydraulic capacitance C correspond respectively to energy losses, inertia and storage. An additional dissipation is introduced and represented in the electrical T-shaped circuit by a hydraulic resistance R_μ to consider the internal processes breaking the thermodynamic equilibrium between the cavitation volume and the surrounding liquid, [1]:

$$R_\mu = \frac{\mu''}{\rho_w g A \mathrm{d}x} \tag{6}$$

where μ'' is the bulk viscosity, ρ_w is the water density, A and dx are the section and the length of the pipe element, respectively, and g is the gravity acceleration.
The model of a pipe of length L is made of a series of n_b elements based on the equivalent scheme of Figure 3. The system of equations relative to this model is then set-up using Kirchoff laws [11], [15].

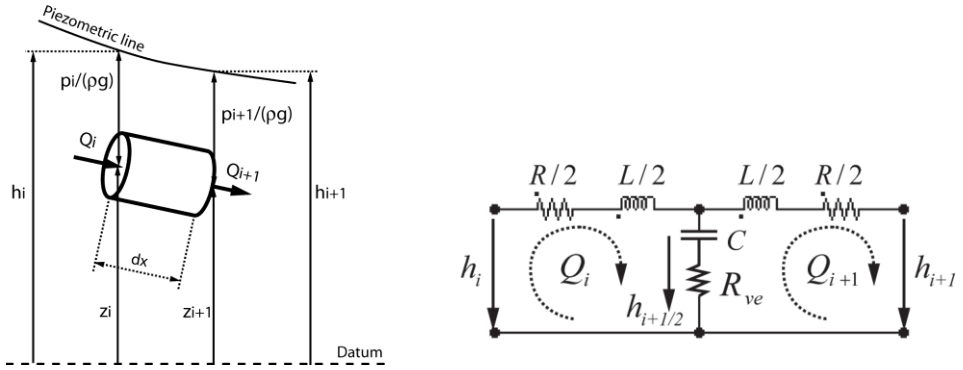


Figure 3. Element of pipe of a length dx and related equivalent circuit.

This electrical analogy of the hydraulic model describes the dynamic behaviour as a first order differential equation system in the matrix form:

$$[A]\frac{d\vec{x}}{dt}+\left[B\left(\vec{x}\right)\right]\cdot\vec{x}=\vec{V}\left(\vec{x}\right) \tag{7}$$

where $[A]$ and $[B(\vec{x})]$ are the state global matrices of dimension $[n \times n]$, $\vec{x}$ and $\vec{V}(\vec{x})$ are respectively the state vector and the boundary conditions vector with n components. The linearization of the system of equation is described by the following relation, where linearized $[B]$ becomes $[B_l]$:

$$[A]\frac{d\cdot\delta\vec{x}}{dt}+[B_l]\cdot\delta\vec{x}=\vec{0} \tag{8}$$

The eigenvalues $s_k = \alpha_k + j\omega_k$ of the system can be calculated from the following characteristic equation:

$$\det\left([\mathrm{I}]s+[A]^{-1}[B_l]\right)=0 \tag{9}$$

By solving this equation, the eigenmodes shapes with related eigenfrequencies can be predicted, [2], [9]. The real part of the eigenfrequency corresponds to the damping α while the imaginary part corresponds to the fluctuation of oscillation ω. Finally, the relative damping is defined by the following equation:

$$\eta=\frac{\alpha}{\sqrt{\omega^2+2\alpha^2}} \tag{10}$$

Besides this modal analysis, the resonance risk assessment can be completed by a forced response analysis. This method allows identifying the contribution of each eigenmode into the system response which depends on the system boundary conditions and the excitation source location [2]. With the forced response method, the equation (4) becomes:

$$[A]\frac{d\vec{x}}{dt}=\left[B\left(\vec{x}\right)\right]\cdot\vec{x}+[C]\cdot\vec{U}+\vec{V}\left(\vec{x},\vec{U}\right) \tag{11}$$

Where $\vec{U}$ is the input vector with p components and $[C]$ is the input matrix of dimension $[n \times p]$. Combining forced response analysis with eigenmodes computation, the system response to hydraulic excitation induced by cavitation vortex rope in the turbine draft tube can be investigated.

3. 1D additional simplified models

3.1. Distributed model including cavitating draft tube

It is possible to estimate the natural frequencies of the hydraulic circuit considering the cavitation developing in the draft tube at part load operation, by modelling the piping system comprised between upstream reservoir and downstream reservoir [11], by an equivalent pipe, see Figure 4, characterized by a total length l_{tot} and an equivalent wave speed a_{equ} given by:

$$l_{tot} = \sum_{i=1}^{n} l_i \qquad\qquad a_{equ} = \frac{l_{tot}}{\sum_{i=1}^{n} \frac{l_i}{a_i}} \tag{12}$$

The total length l_{tot} and an equivalent wave speed a_{equ} shall include the draft tube length l_{DT} and the draft tube wave speed a_{DT}. The corresponding natural frequencies are then given by:

$$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k \quad ; \quad k = 1, 2, 3... \tag{13}$$

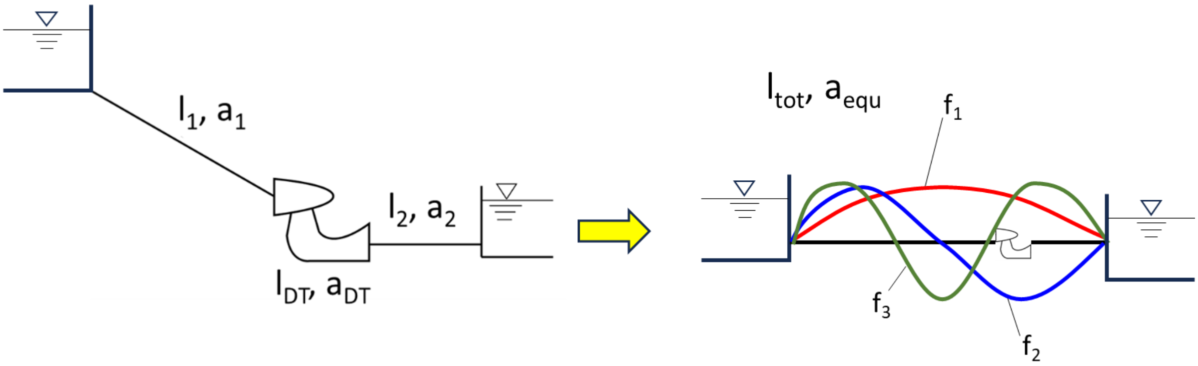


Figure 4. Hydraulic system modelled with an equivalent pipe and corresponding modes shapes for the 3 first natural frequencies.

3.2. Lumped models

Figure 5 presents the simplified hydroacoustic model of the frictionless cavitating draft tube composed of the cavitation compliance C_{DT} and of the draft tube inductance L_{DT} given by:

$$C_{DT} = \frac{l_{DT} \cdot g \cdot \bar{A}_{DT}}{a_{DT}^{2}} \qquad\qquad L_{DT} = \frac{l_{DT}}{g \cdot \bar{A}_{DT}} \tag{14}$$

Where:

l_{DT} : equivalent draft tube length (m)

$\bar{A}_{DT}$: mean draft tube cross section area (m^2)

a_{DT} : mean cavitating draft tube wave speed (m·s^{-1})

g : acceleration due to gravity (m·s^{-2})

Where, the mean cross section area of the draft tube can be calculated based on equivalent inductance approach enabling to calculate the equivalent cross section area as follows:

$$A_{equ} = \frac{l_{tot}}{\sum \frac{l_i}{A_i}} \tag{15}$$

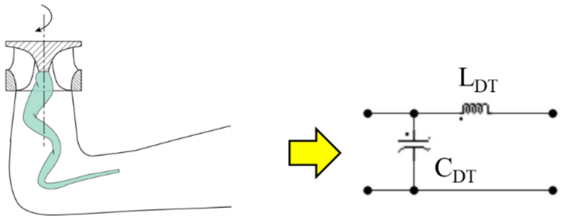


Figure 5. Cavitating draft tube modelled with cavitation capacitance and draft tube inductance.

Assuming that the draft tube is connected downstream to an infinite reservoir and that the turbine runner represents an infinite hydraulic resistance upstream the draft tube, the first natural frequency of this system is given by Jacob [7]:

$$f_o = \frac{1}{2\cdot\pi}\frac{1}{\sqrt{L_{DT}\cdot C_{DT}}} = \frac{1}{2\cdot\pi}\frac{a_{DT}}{l_{DT}} \tag{16}$$

When the draft tube is connected to a downstream tailrace pipe, see Figure 6, the inertia of this pipe is dominant and the corresponding simplified model includes the draft tube cavitation compliance C_{DT} and the tailrace pipe inductance L_{TR} given by:

$$L_{TR} = \frac{l_{TR}}{g\cdot A_{TR}} \tag{17}$$

Where:

l_{TR} : tailrace pipe length (m)

A_{TR} : tailrace pipe cross section area (m^2)

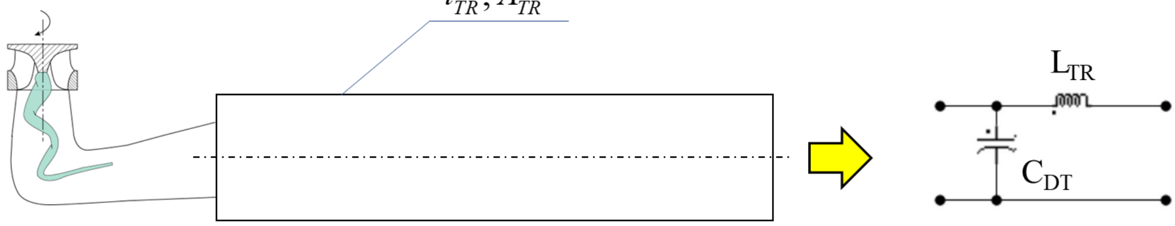


Figure 6. Cavitating draft tube with long tailrace tunnel modelled with cavitation capacitance and tailrace tunnel inductance.

Assuming again that the tailrace pipe is connected downstream to an infinite reservoir and that the turbine runner represents an infinite hydraulic resistance upstream the draft tube, the first natural frequency of this system is given by Dörfler [5]:

$$f_o = \frac{1}{2\cdot\pi}\frac{1}{\sqrt{L_{TR}\cdot C_{DT}}} = \frac{1}{2\cdot\pi}\frac{a_{DT}}{\sqrt{l_{DT}\cdot l_{TR}\frac{\overline{A}_{DT}}{A_{TR}}}} \tag{18}$$

The tailrace pipe may also include the inertia characteristics of the draft tube.

4. Examples of application

The calculation of the system natural frequencies based on formulas (13), (16) and (18) above is applied to three different hydraulic systems to evaluate part load resonance risk and compared with detailed calculation results. These hydraulic systems presented in Figure 7 includes:

- **Hydraulic system 1:** an upper reservoir, a penstock, a Francis turbine, a draft tube, a downstream reservoir;
- **Hydraulic system 2:** same as hydraulic system 1 with adjunction of a tailrace pipe with same diameter as the draft tube;
- **Hydraulic system 3:** same as hydraulic system 1 with adjunction of a tailrace pipe with different diameter than the draft tube.

The hydraulic system 1 to 3 are modelled in SIMSEN to compute reference eigenvalues to compare with simplified analytical methods. The parameters of the hydraulic systems 1 to 3 are summarized in Table 1. According to the Francis turbine rotational speed, the vortex rope pressure fluctuation frequency is expected to be between 2.5 Hz and 5 Hz. If one of the hydraulic system natural frequencies fall into this interval, resonance may take place.

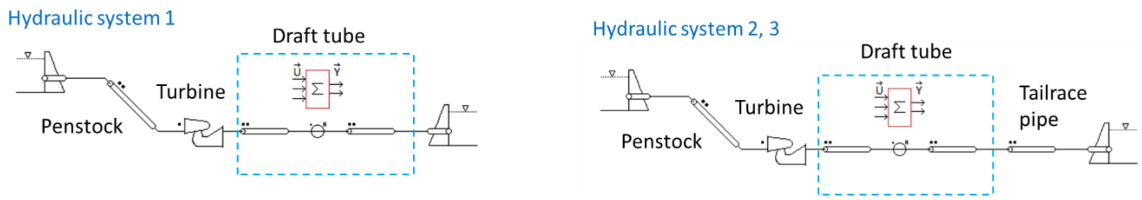


Figure 7. Hydraulic system 1 (left) and hydraulic systems 2 and 3 (right).

Table 1 Parameters of the hydraulic systems 1, 2 and 3.

Penstock		Turbine		Draft tube		Tailrace pipe	
L	300 m	P_n	5 MW	L	10 m	L	100 m
D	1.2 m	Q_n	5 m^3/s	D	1.2 m	D	1.2 or 2 m
a	1250 m/s	H_n	100 m	a	50 or 100 m/s	a	1250 m/s
λ	0.012 -	N_n	750 min^{-1}	λ	0.012	λ	0.012
		D_{ref}	0.864 m				
		N_q	53				
		f_{exct}	2.5~5 Hz				

The parameters of the equivalent pipe of the hydraulic systems 1, 2 and 3 computed with equations (12) and (15) for draft tube wave speed values of 50 m/s and 100 m/s are provided respectively in Table 2, Table 4 and Table 6. The related natural frequencies computed based on formulas (13), (16) and (18) which are compared with the results of numerical eigenvalue calculation as described in chapter 2 are provided respectively in Table 3, Table 5 and Table 7 with the corresponding errors. The range of prototype draft tube wave speed selected for this analysis has shown to provide rather good results with site measurements for couple of test cases [6], [16]. The wave speed can also be deduced from the reduced cavitation compliance [4] or deduced from reduced scale model test if available [9]. The first natural frequency f_0 is computed with formula (16) for hydraulic system 1 and with formula (18) for hydraulic systems 2 and 3. The natural frequencies f_1 to f_6 are then calculated according to formula (13). Resonance have already been found up to the 5th hydraulic system natural frequencies [6], [10], [16], this is why the first 6 natural frequencies have been computed. In principle the higher the order, the higher the relative damping, reducing the amplitude of the pressure fluctuations along the hydraulic circuit resulting to a forced excitation.

The analysis of the errors obtained for the natural frequencies f_2 to f_6 shows for each hydraulic system a rather good agreement between the analytical approach and the eigenvalue calculation with a maximum error of 14%. Regarding the natural frequencies f_0 and f_1, which are both compared with the first natural frequency obtained by the eigenvalue calculation, it could be noticed that for the hydraulic system 1 without tailrace pipe, better agreement is found with the natural frequency f_1 computed with formula (13) even if the frequency f_0 computed with formula (16) gives the right order of magnitude. However, for the hydraulic systems 2 and 3, very good agreement is found for f_0 with the formula (18) while formula (13) leads to very large differences for f_1. Therefore, in practice, it is recommended to compute both natural frequencies f_0 (lumped model) and f_1 (distributed), and also the natural frequencies up to f_6, and to check possible resonance risk. If such a risk is identified, it is recommended to perform additional investigations with the more detailed approach of chapter 2 such as forced response analysis, in order for example to anticipate possible mitigation measures during the design phase, which can be also consolidated after reduced scale model tests.

In Table 3, Table 5 and Table 7, the natural frequencies which can potentially lead to resonance with draft tube vortex rope excitation in the range 2.5 Hz to 5 Hz are highlighted in bold. The hydraulic system 1 is the simplified model of an existing prototype unit which is known to suffer from part load resonance between the cavitating vortex rope and the 3rd natural frequency of the hydraulic system. Site measurements showed a resonance frequency value of 3.8 Hz, which is in good agreement with the results from Table 3 where the third natural frequency of the system is found between 3.41 Hz and 4.41 Hz with the proposed analytical approach.

Table 2 Parameters of the equivalent pipe of the hydraulic system 1.

System1	L (m)	a (m/s)	D (m)
Penstock	300	1250	1.2
Draft tube	10	50 100	1.2
Equivalent pipe	310	704.5 911.8	1.2

Table 3 Estimation of the natural frequencies f_0 to f_6 of the hydraulic system 1 based on formulas (13) and (16) and comparison with results obtained with 1D numerical eigenvalue calculation and corresponding errors.

System1	Analytical calculation		1D Numerical calculation		Error	
	a_DT = 50 m/s	a_DT = 100 m/s	a_DT = 50 m/s	a_DT = 100 m/s	a_DT = 50 m/s	a_DT = 100 m/s
f_0 (Hz)	0.80	1.59	1.24	1.94	-36%	-18%
f_1 (Hz)	1.14	1.47			-8%	-24%
f_2 (Hz)	2.27	**2.94**	2.09	2.61	9%	13%
f_3 (Hz)	**3.41**	**4.41**	3.67	4.17	-7%	6%
f_4 (Hz)	**4.55**	5.88	4.18	6.10	9%	-4%
f_5 (Hz)	5.68	7.35	5.99	7.38	-5%	0%
f_6 (Hz)	6.82	8.82	6.15	8.24	11%	7%

Table 8 presents the pressure mode shape obtained by eigenvalue and eigenvector calculation for the 3 first natural frequencies f_1, f_2 and f_3 of the hydraulic system 1 and 2 featuring respectively 1, 2 or 3 pressure antinodes for draft tube wave speed of 50 m/s. It could be noticed that the modes f_2 and f_3 corresponds to elastic pressure mode shapes. For the first natural frequency, the pressure mode shape of the hydraulic system 1 features also an elastic mode shape, while the hydraulic system 2 is characterized by a rigid column mode shape, where the pressure along the tailrace is proportional to the length, similar to the case of surge tank mass oscillation between the tailrace pipe and the draft tube compliance [12]. It explains why the value of natural frequency is better captured with formula (18 (lumped model)) than with formula (13 (distributed model)).

Table 4 Parameters of the equivalent pipe of the hydraulic system 2.

System2	L (m)	a (m/s)	D (m)	A (m^2)
Penstock	300	1250	1.2	1.13
Draft tube	10	50 100	1.2	1.13
Tailrace	100	1250	1.2	1.13
Equivalent pipe P + DT + TR	410	788.5 976.2		

Table 5 Estimation of the natural frequencies f_0 to f_6 of the hydraulic system 2 based on formulas (13) and (18) and comparison with results obtained with 1D numerical eigenvalue calculation and corresponding errors.

System2	Analytical calculation		1D Numerical calculation		Error	
	a_DT = 50 m/s	a_DT = 100 m/s	a_DT = 50 m/s	a_DT = 100 m/s	a_DT = 50 m/s	a_DT = 100 m/s
f_0 (Hz)	0.25	0.50	0.27	0.55	-7%	-8%
f_1 (Hz)	0.96	1.19			256%	116%
f_2 (Hz)	1.92	2.38	2.04	2.10	-6%	13%
f_3 (Hz)	**2.88**	**3.57**	2.53	4.05	14%	-12%
f_4 (Hz)	**3.85**	**4.76**	4.12	4.88	-7%	-2%
f_5 (Hz)	**4.81**	5.95	4.87	6.18	-1%	-4%
f_6 (Hz)	5.77	7.14	5.16	6.36	12%	12%

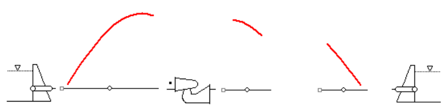
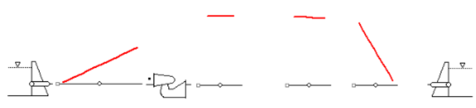
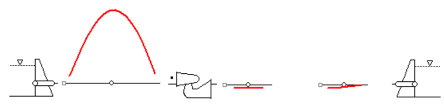
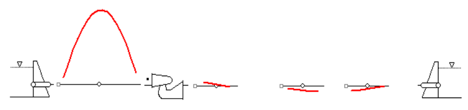
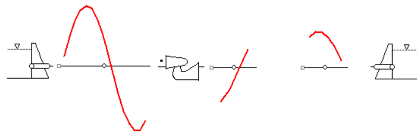
Table 6 Parameters of the equivalent pipe of the hydraulic system 3.

System3	*L* (m)	*a* (m/s)	*D* (m)	A (m^2)
Penstock	300	1250	1.2	1.13
Draft tube	10	50 100	1.2	1.13
Tailrace	100	1250	2.0	3.14
Equivalent pipe P + DT + TR	410	788.5 976.2		

Table 7 Estimation of the natural frequencies f_0 to f_6 of the hydraulic system 3 based on formulas (13) and (18) and comparison with results obtained with 1D numerical eigenvalue calculation and corresponding errors.

System3	Analytical calculation		1D Numerical calculation		Error	
	a_DT = 50 m/s	a_DT = 100 m/s	a_DT = 50 m/s	a_DT_max	a_DT = 50 m/s	a_DT = 100 m/s
f_0 (Hz)	0.42	0.84	0.42	0.81	0%	4%
f_1 (Hz)	0.96	1.19			129%	47%
f_2 (Hz)	1.92	2.38	2.04	2.10	-6%	13%
f_3 (Hz)	**2.88**	**3.57**	2.55	4.03	13%	-11%
f_4 (Hz)	**3.85**	**4.76**	4.12	4.72	-7%	1%
f_5 (Hz)	**4.81**	5.95	4.84	6.14	-1%	-3%
f_6 (Hz)	5.77	7.14	6.16	6.55	-6%	9%

Table 8 Pressure mode shape obtained by eigenvalue and eigenvector calculation for the 3 first natural frequencies f_1, f_2 and f_3 of the hydraulic system 1 and 2 for the draft tube wave speed of 50 m/s.

	Hydraulic system 1	**Hydraulic system 2**
f_1		
f_2		
f_3		

5. Limitation of the models

The proposed methodology has given good results for a simple hydraulic system with single branch hydraulic layout. For systems with parallel branches, as a first approach, the parallel branches can be modelled by a single branch with equivalent parameters to obtain a first order of magnitude of the system natural frequencies. However, the real system will feature much more complex and numerous eigenvalues, due to the hydraulic system asymmetry, see [16], change of diameters and all bifurcations which are neglected in a single branch approach.

6. Conclusions

This paper is summarising some simple analytical approaches to estimate hydroacoustic natural frequencies of systems including cavitating draft tubes at part load operation. Such system may be subject to system resonance in case where the dominant excitation frequency of the cavitating vortex rope matches one of the system natural frequencies. The simplified methodology considered in this paper provides reasonably good results to identify potential risk of resonance at early stage of the project.

The main outcome of this paper is the confirmation that the simplified analytical approach considered in the present paper is valid to perform a first part load vortex rope resonance risk assessment at feasibility stage already. In case such risk is identified at early stage of a project and confirmed with more detailed approach, it will allow to specify specific reduced scale model tests to test and anticipate possible mitigation measures.

Therefore, if the analytical approach points out possible resonance risk, it is recommended to perform detailed eigenvalue and eigenvector analysis to improve the accuracy of the evaluation. The eigenvalues enable to deduce the relative damping which may apply on risky mode shapes which frequencies lays within the range of 0.2 to 0.4 times the runner rotational speed. The lower the relative damping, the higher amplification of pressure fluctuations. Nevertheless, the exact location of the excitation source within a given eigenmode has an important impact on the resulting dynamic amplification factor. Therefore, forced response calculation taking into account pressure fluctuation excitation source located in the draft tube enable to refine the resonance risk assessment and evaluate the corresponding dynamic amplification factor along the waterway. When reduced scale model tests are performed, exact excitation frequencies are known and corresponding amplitudes may be deduced, [3], [4], in order to quantify pressure fluctuation amplitudes along the waterway.

The present simplified approach, together with more advanced part load resonance evaluation methods have been included as the Annexe E.3 of the new IEC Technical Specification TS62882 Ed1, 2020, [18], entitled “Hydraulic machines – Francis turbine pressure fluctuation transposition” issued in 2020-09. This IEC technical specification describes the most common pressure fluctuations phenomena encountered in Francis turbine and proposes ways to transpose the results from reduced scale model to prototype, as well as some mitigation measures, which are certainly of interest for the hydro industry.

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